



Direct Kinematics	Analysis
	<p>We observe that:</p> $x = L1 \cos(\theta_1) + L2 \cos(\theta_1 + \theta_2)$ $y = L1 \sin(\theta_1) + L2 \sin(\theta_1 + \theta_2)$ <p>which is the direct kinematics analysis for the robot for its two first articulations</p> <p>We set $\varphi = \theta_1 + \theta_2$</p>
Inversed Kinematics	Analysis
	<p>From Pythagora's theorem:</p> $OB^2 = x^2 + y^2$ <p>Moreover</p> $OB^2 = (L1 + L2 \cos(\theta_2))^2 + (L2 \sin(\theta_2))^2 = L2^2 \sin^2(\theta_2) + L1^2 + L2^2 \cos^2(\theta_2) + 2 L1 L2 \cos(\theta_2)$ $\Leftrightarrow x^2 + y^2 - L1^2 - L2^2 (\cos^2(\theta_2) + \sin^2(\theta_2)) = 2 L1 L2 \cos(\theta_2)$

So we take:

$$x^2 + y^2 - L1^2 - L2^2 = 2 L1 L2 \cos(\theta_2) \Leftrightarrow \cos(\theta_2) = \frac{x^2 + y^2 - L1^2 - L2^2}{2 L1 L2}$$

$$\theta_2 = \arccos\left(\frac{x^2 + y^2 - L1^2 - L2^2}{2 L1 L2}\right)$$

Now for θ_1 :

$\varphi = \theta_1 + \theta_2 \Leftrightarrow \theta_1 = \varphi - \theta_2$ but what if we don't know φ ? Well, in that case:

$$\begin{array}{l} \text{ABO' triangle:} \\ \sin(\theta_2) = \sqrt{1 - \cos(\theta_2)^2} \end{array}$$

$$\begin{array}{l} \text{BOA corner of BOO' triangle:} \\ \text{BOA corner} = \arctan\left(\frac{BO'}{L1 + AO'}\right) = \arctan\left(\frac{L2 \sin(\theta_2)}{L1 + L2 \cos(\theta_2)}\right) \end{array}$$

Let's say that $\text{BOA corner} + \theta_1 = \gamma$ so:

$$\theta_1 = \gamma - \text{BOA corner} = \arctan\left(\frac{y}{x}\right) - \arctan\left(\frac{L2 \sin(\theta_2)}{L1 + L2 \cos(\theta_2)}\right)$$