SCIENCES SORBONNE UNIVERSITÉ
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3T622 - Laboratory Internship

Convection

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Scientific question:
Develop an analog model to describe convection movements within the Earth's mantle. The aim is to improve the experiment to measure critical parameters to estimate the critical Rayleigh number marking the beginning of convection. To verify that our experiments are correct, we test them with a fluid whose parameters can be theoretically determined, engine oil. Then we can apply these experiments to fluids with characteristics closer to those of the mantle, such as gel wax.
Objectives: calculate Rayleigh's number and check from experiments the 3 coefficients of the Rayleigh formula.

Theory and Programs

Description of the theory used to set up experiments and programs developed for data recovery and processing.

The Rayleigh number

$$R_a = \frac{\alpha g \Delta T h^3}{k\upsilon}$$

Determination of the critical Rayleigh number from which fluid convection begins,

➢ Experiment 1: Determination of Thermal Expansion α

At the beggining, we have this equation : $\rho = \rho 0(1-\alpha\Delta T)$

Expression of thermal expansion :

$$\alpha = \frac{Sdh(T-T_0)}{Sdh+V_0}$$

From this formula, we must find the temperature coefficient from the density and the temperature difference. It is necessary to know the section of the tube and the initial volume of the oil in the balloon.
The oil is heated from 20 to 60°C.The temperature is reported (every 1°C) with the corresponding tube height. We obtain a graph where we do a linear adjustement so we can calculate the slope and find α.

➢ Experiment 2: Determination of Kinematic Viscosity υ

From the falling velocity of the ball in the fluid we can go back to the kinematic viscosity by the following equations:

Stokes' Law : $v = \frac{2r^2 g\Delta\rho}{9\mu}$

Expression of dynamic viscosity as a function of kinematic viscosity:

$$\mu = \rho\upsilon$$

Writing a program on Matlab to process videos:
Picture-by-picture pointing system on the time interval of the log fall. Each point corresponds to the position of the ball, the first two points make it possible to determine the total distance to be covered, which is equal to the height of the fluid in the specimen. We obtain a graph with the ball positions on the y-axis and the time on the abscissa. When the ball speed stabilizes, we can make a linear adjustment that gives us a line whose slope is the ball speed.

➢ Experiment 3: Determination of Thermal Diffusivity κ

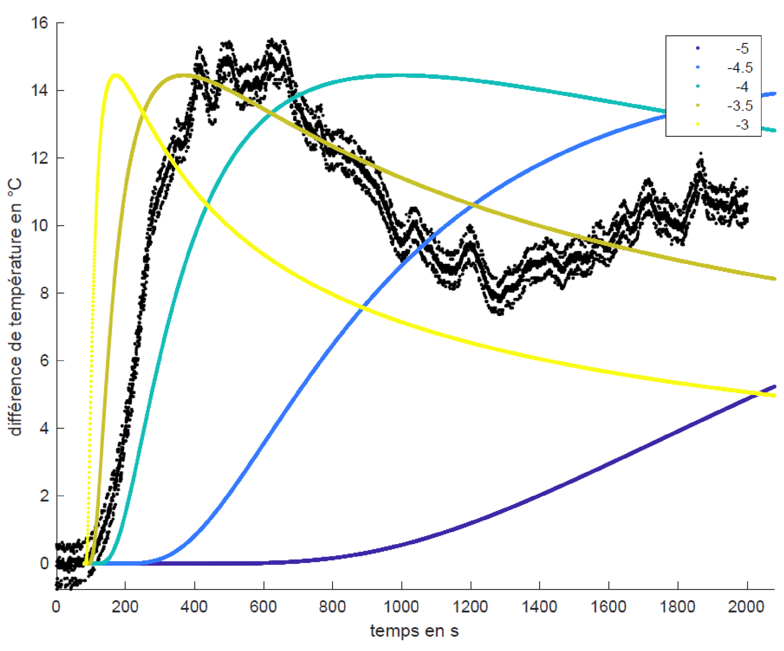
Expression of thermal diffusivity:

$$\kappa = \frac{k}{\rho C_p}$$

Program to determine the temperature recorded by a probe from the digital data it provides to a computer. Coupling of three probes in the program in order to determine the ΔT necessary for the calculation of K.

Program to determine k:.

We will create two vectors: x and t,forming a grid. Then, for a given range of kappa, we will fill a table of values of temperature as a function of the distance x. Knowing our Δx in our experiment, we will then be able to determine two values of T.Finally, we will plot ΔT as a function of t (Figure, black curve) that we will compare with theoretical data of log of kappa (colored curves). Thus, we can approximate a kappa.



Graphe 4: ΔT as fonction of x

Experiences Implemented

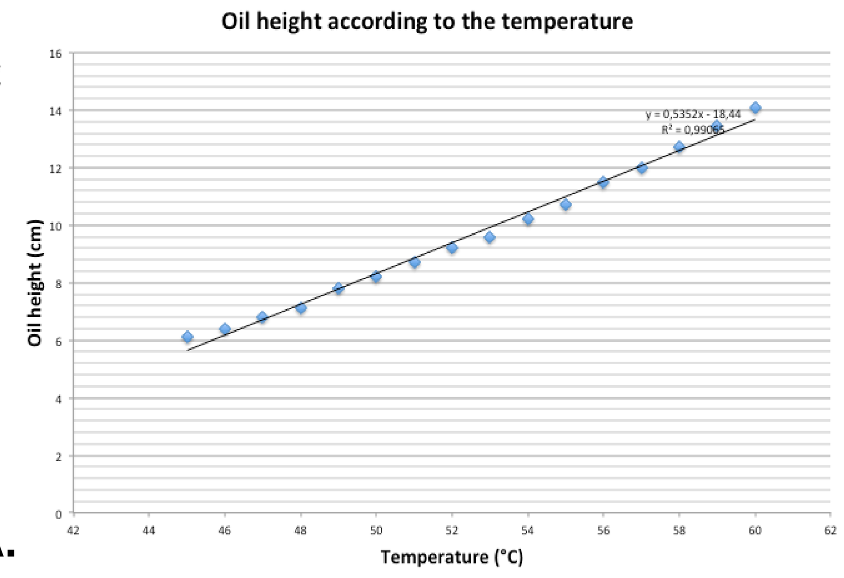
➢ Determination of Thermal Expansion α

Materials used : thermometer, balloon filled with oil, glass tube, plug, oil (Figure 1 A.)
When we obtain the graphic (Graphic 1 B.), we calculate alpha for a temperature difference equal to 1°C. The average of alpha values will give an approximate value of alpha.

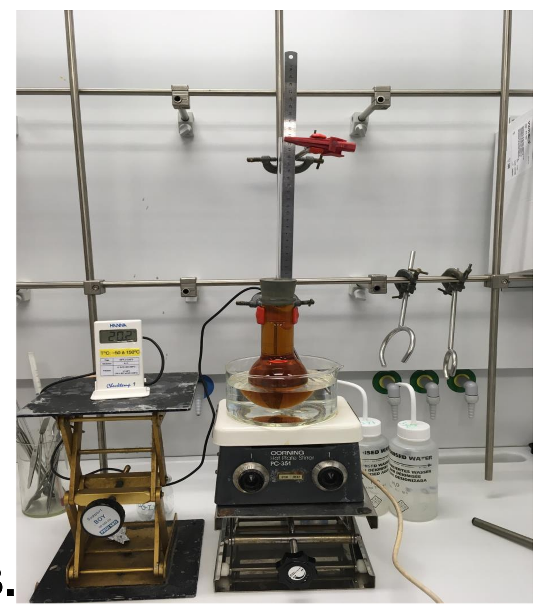
Results: For a ΔT = 1°C
α=1,12.10^{-3} K-1 ±7.10^{-4}

A. Graphic 1 : Oil height according to the temperature

B . Figure 1 : Experiment to derterminate Thermal Expansion



A.

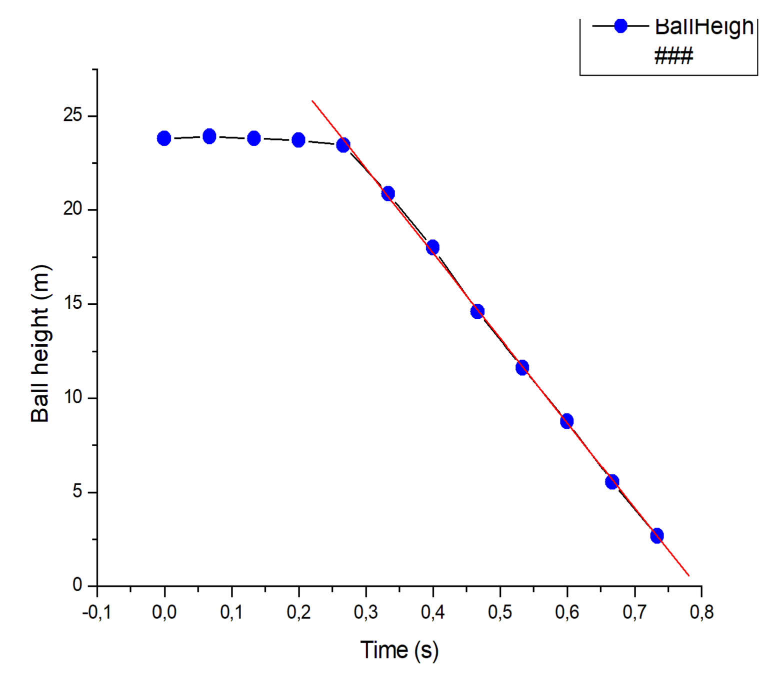


B.

➢ Determination of Kinematic Viscosity υ

Principle of experience: Drop a ball whose dimensions, density and mass are known in a fluid (rapeseed oil, motor oil, gel wax), take a video of its fall, by image processing recover the kinematic viscosity of the fluid from the drop velocity of the ball
Equipment used: Graduated cylinder, aluminum ball, our 3 fluids, camera, hot plate (gel wax),pliers

Results: $\upsilon = 348 \pm 16\ (1\sigma) mm^2/s$



Graphic 2 : Determination of speed, log height as a function of time

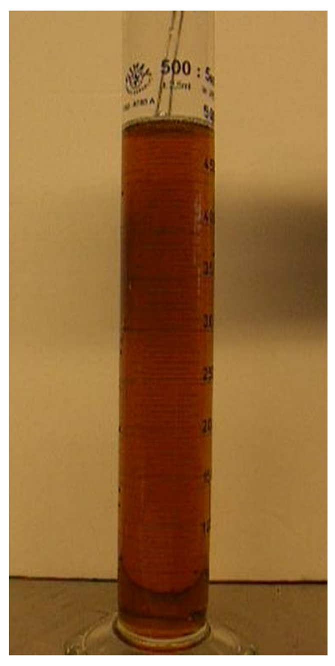
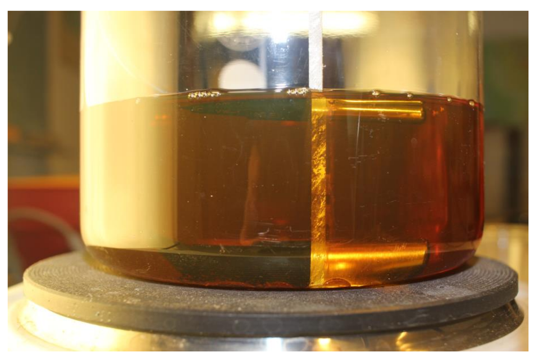


Figure 2 : Experiment to derterminate kinematic viscosity

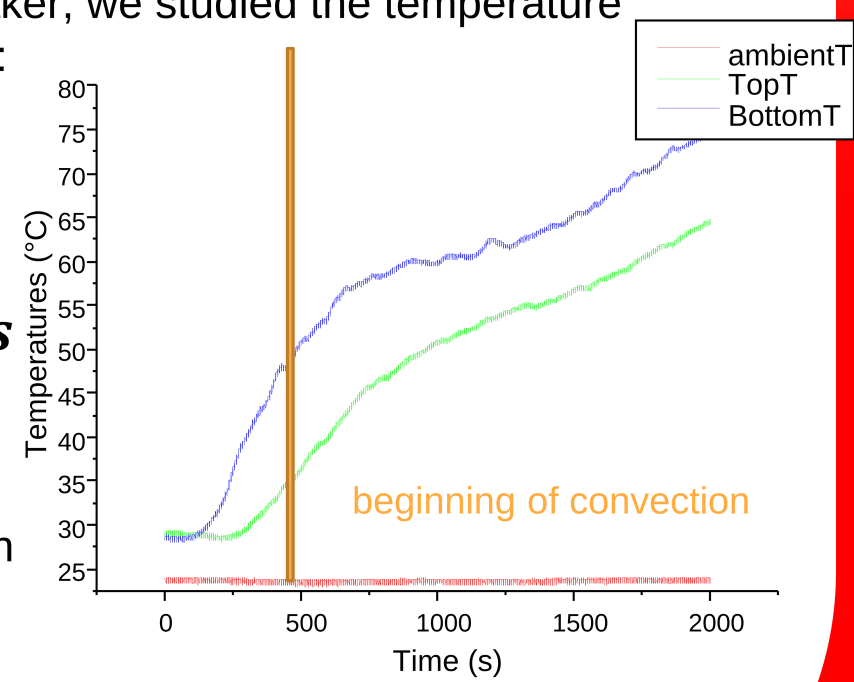
➢Determination of thermal diffusivity coefficient κ

In a 3L beaker, we introduced 1L of motor oil (Figure 3). Then we placed 3 probes, one at the low end of the oil, at the bottom of the beaker, another at the upper limit of the oil, close to the surface and the last one on the table in order to measure the ambient temperature (Figure4).Subsequently,we placed this device on a hotplate.



Figures 3 and 4

We heated the motor oil so that we could observe convection cells. Thanks to the 2 probes in the beaker, we studied the temperature between both. Here is the graph :
With these data, we were able to determine K by using a program on Matlab.
Results:
$\kappa = 2.10^{-4} \pm 1.\ 10^{-4}\ m^2/s$
Note: our hotplate does not heat at a constant temperature and heat flux, which distorts experiment. Moreover, there is an underestimation of the real ΔT because the temperature measured in bottom is not that emitted by the hotplate.



Graphe 3: Temperatures as fonction of time

It would have been necessary to measure this temperature with a thermo couple inserted in the glass of the beaker.

Implication at geological scales

We find the phenomenon of convection within the Earth's mantle but also inside the outer core of the Earth. This is the main process leading to plate tectonics. Moreover, we find convection in air mass displacements as well as in oceanic movements.
But the determination of the Rayleigh number of the mantle is more difficult to put in place. Indeed, h, α, υ and k are easily identifiable but g is not very quantifiable because it varies a lot and we do not know the temperature gradient of the mantle, so delta T is difficult to assess as well.

Experimental values	Theoretical values
α = 1,12.10^{-3} K-1 ±7.10^{-4}	α = 0,00062
~	g=9,81 (m/s)
ΔT = 20± 5 (C°)	~
h = 0,053	~
κ = $2.10^{-4} \pm 1.\ 10^{-4}$	κ = 1,43.10^{-4}
$\upsilon = 348 \pm 16\ (1\sigma) mm^2/s$	$\upsilon \approx 500 mm^2/s$

Calculation of the experimental Rayleigh number

From our experimental values and taking into account the uncertainties we deduce from them R_a:

$$R_{a\ min} < R_{a\ exp} < R_{a\ max}$$
$$110 < 623{,}1 < 2500$$
$$R_{a\ moyen} = 584$$

Conclusion:

Through our experiments we have succeeded in determining the coefficients allowing us to calculate a critical Rayleigh number close to the theoretical number for this height. The next step is therefore to use these experiments to determine the coefficients and critical Rayleigh number for a fluid with characteristics close to those of the mantle, for example gel wax.

Sources:

- CARACTERISTIQUES DES HUILES.doc
- http://www.ardumotive.com/how-to-use-the-ds18b20-temperature-sensor-en.html
- http://fablab.sorbonne-universites.fr/wiki/doku.php?id=wiki:projets:convection_geosciences

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